

POLYMAKE Exercises

Michael Joswig

Bernd Sturmfels

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Technical Advice: look up www.polymake.org. For software download and installation procedures click “Download”. Precompiled versions available for Ubuntu 11.04 and MacOS 10.6 (sorry, no MacOS 10.7 yet).

1. Eliminate the unknown z from the system of linear inequalities

$$0 \leq x + y - 2z \leq 1 \text{ and } 0 \leq x - 2y + z \leq 1 \text{ and } 0 \leq -2x + y + z \leq 1.$$

2. Consider all vectors in \mathbb{R}^6 obtained from $(0, 0, 1, 1, 2, 2)$ by permuting coordinates, let P denote their convex hull. Write the set P as the solution set of a system of linear inequalities in six unknowns.
3. The *hexadecachoron* is the convex hull of the even vertices of the 4-cube:

$$P = \text{conv} \left\{ \begin{array}{l} (0, 0, 0, 0), (0, 0, 1, 1), (0, 1, 0, 1), (0, 1, 1, 0), \\ (1, 0, 0, 1), (1, 0, 1, 0), (1, 1, 0, 0), (1, 1, 1, 1) \end{array} \right\}.$$

Determine the f -vector of P . Is P simple? Is P simplicial? Draw a Schlegel diagram. How about the analogous polytope in 5 dimensions?

4. Let P denote the convex hull in \mathbb{R}^3 of the nine points with coordinates (i, i^2, i^3) for $i = -4, -3, -2, -1, 0, 1, 2, 3, 4$. Compute the volume of P .
5. Select eight points at random on the unit sphere \mathbb{S}^3 in \mathbb{R}^4 . Determine (experimentally) the expected number of facets of their convex hull.
6. How many 4×4 -matrices with non-negative integer entries and zeros on the diagonal have all row sums and all column sums equal to m ? Determine this number for $0 \leq m \leq 7$ and formulate a conjecture.

7. Let Δ be the simplicial complex of stable sets in the Petersen graph. Determine all the homology groups of Δ .
8. Compute the Newton polytope of the $2 \times 2 \times 2$ -hyperdeterminant.
9. Draw a three-dimensional tropical polytope that is not a polytope.
10. Construct a polyhedral fan with ten rays that represents a *smooth* three-dimensional compact toric variety. Is your toric variety projective?
11. Determine the f-vector of the matroid polytope of the Fano plane.
12. Let T be the *snowflake tree*, with six leaves, and all nine edges having length one. Determine the tropical plane in \mathbb{TP}^5 whose tropical Plücker coordinates are the subtree weights on triples of leaves.
13. Identify the edges in the tree T (from the exercise above) with basis vectors in \mathbb{R}^9 . Compute the convex hull of the 15 incidence vectors of paths between pairs of leaves.
14. Let H be a 4×4 -Hankel matrix with indeterminate entries. Compute the Newton polytope of the product of all 3×3 -subdeterminants of H .
15. Construct 50 combinatorially distinct 4-dimensional polytopes, each of which has precisely 30 vertices and is neither simple nor simplicial.